and make a second transformation to the variables s, v, w defined by

$$s=s\,,\quad u=vs\,,\quad t=wvs\,. \tag{A7}$$

It can be shown in a straightforward manner that the Jacobian of the transformation

$$\partial(s,u,t)/\partial(s,v,w)=vs^2\,, \tag{A8}$$

whence

$$F_B(q^2)=(N^2/2)\int_{-1}^{+1}dy\int_0^1 dv\int_0^1 dw\int_0^\infty ds\,\exp\{s[-\kappa(1+v)+(iqy/3)(1+v^2(w^2-1))^{1/2}]\}s^5v^2(1-w^2v^2)\,.$$

Integrations with respect to s and y are simple and give

$$F_B(q^2)=12N^2\int_0^1 dv\int_0^1 dw\frac{v^2(1-w^2v^2)\{[\kappa(1+v)+(iq/3)\{1+v^2(w^2-1)\}^{1/2}]^5-[\kappa(1+v)-(iq/3)\{1+v^2(w^2-1)\}^{1/2}]^5\}}{(iq/3)\{1+v^2(w^2-1)\}^{1/2}[\kappa^2(1+v)^2+(q^2/9)\{1+v^2(w^2-1)\}]^5}\,. \tag{A9}$$

After some simplification this gives Eq. (5).

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Nuclear Core Polarization Effect on Beta Decay*

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It is shown that the neutron-hole and proton correlations play a significant role for beta decays in heavier nuclei. The effective coupling constant of beta decay is estimated by using a soluble model and experimental knowledge of (p,n) reactions. Systematics of f_0t values are re-examined qualitatively.

1. INTRODUCTION

MANY attempts[1–4] have already been made for understanding the f_0t values of beta decays. Among others the so-called blocking effect in the pairing model[3,4] can explain the relative f_0t values of some isotopes successfully. However, these current nuclear theories seem still imcomplete in explaining the absolute magnitude of beta transitions.

The purpose of this paper is to call attention to the neutron-hole-proton (in short, $\bar{n}$-p) correlation effects. The study[5,6] of $\bar{n}$-p correlations[7] was motivated by the experimental discovery[8] of isobaric resonances[9,10] in $(p$-$n)$ reactions. Existence of the well-defined isobaric states is very important for beta decay theories, because the transition amplitude to the isobaric state exhausts the sum rule for $\int 1$. In the previous note[5] it was shown that the isobaric state can be interpreted as a coherent

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mixture of the $(N-Z)$ $\bar{n}$-p states, and this interpretation led us to the other result[6] that the transition strength of Gamov-Teller type $\int\boldsymbol{\sigma}$ should also be concentrated in the higher excited states, being pushed up by $\bar{n}$-p correlations.

In this paper, simple soluble models are discussed, which are based on the Fermi gas model. The mathematical structure of the models is completely analogous to that used by Brown and others[11] for the discussion of electromagnetic transitions. The argument given in Sec. 2.1 is a simplified version of the previous ones[5,6] using the shell models. This simple model will make clear how the energy of the core polarized state is related to the strength of residual interactions. In Sec. 2.2, the effect of the nuclear core polarization to the beta decay of an outside nucleon is discussed by using the Fermi gas model and taking into account the pairing energies. This model will make it clear that the surplus $(N-Z)$ neutrons are important for determining the absolute magnitude of beta decay.

In Sec. 3, the formulas for nuclear matrix elements of residual interactions are given in the case of the $L-S$ coupling shell model, which are applicable to the allowed and forbidden transitions. By semiphenomenological arguments the parameters in the expression of the effective coupling constant are fixed, and the theoretical results are compared with the observed $\log f_0 t$ values. It is concluded that the existence of the hindrance effect due to the nuclear core polarization is consistent with the experimental knowledge.

2. FERMI GAS MODEL OF NUCLEAR CORE POLARIZATION

Suppose that a neutron with momentum $\mathbf{p}_i$ decays into a proton with $\mathbf{p}_f=\mathbf{p}_i+\mathbf{q}$ outside the Fermi spheres $(N\neq Z)$. We shall discuss how this transition could be hindered when residual interactions are switched on.

The assumption that the outside nucleon has a fixed linear momentum is not essential since the momentum distribution in the initial and final states can be taken into account *a posteriori*. On the other hand, another assumption, that the doubly closed core is treated as a Fermi gas, necessarily means the omission of effects of the nuclear core surface. However, for the sake of simplicity, this model seems to be the best starting point to understand the $\bar{n}$-p correlation effects.

2.1. Polarization of the Closed Core

First, let us discuss the collective states, which can be made by adding a $\bar{n}$ and a p to the doubly closed core $(N\neq Z)$. These are closely related to the resonance peaks in (p,n) reaction.[5]

We shall denote the wave function of the doubly closed core as $|0\rangle$. According to the assumptions, the Fermi momenta have different values, p_{Fp} and p_{Fn}, for the proton and the neutron, respectively. The annihilation operator of a proton (neutron) will be written as a (b). Then an unperturbed $\bar{n}$-p pair state with total momentum $\mathbf{q}$, total spin angular momentum S and its z component S_z is given by

$$|\mathbf{k}+\mathbf{q},\mathbf{k};S(S_z)\rangle=\sum_{\mu_p,\mu_n}(-)^{\frac{1}{2}-\mu_n}\langle\tfrac{1}{2}(\mu_p)\tfrac{1}{2}(-\mu_n)|S(S_z)\rangle \times a^\dagger_{\mathbf{k}+\mathbf{q},\mu_p}b_{\mathbf{k},\mu_n}|0\rangle, \quad (1)$$

where μ_p and μ_n represent the z components of the proton and neutron spins, respectively. Because of the Pauli principle, we have the restrictions, $|\mathbf{k}|<p_{Fn}$ and $|\mathbf{k}+\mathbf{q}|>p_{Fp}$. If $0<q<p_{Fn}-p_{Fp}$, the $\bar{n}$-p pairs can take $(N-Z)$ values of $\mathbf{k}$ and spin directions. In particular, when the relation $q\ll p_{Fn}-p_{Fp}$ is satisfied, as in the usual beta decay, all the $(N-Z)$ states can be regarded as degenerate. Namely, if we write the energy of $|\mathbf{k}+\mathbf{q},\mathbf{k};S(S_z)\rangle$ as $E_{\mathbf{k}+\mathbf{q},\mathbf{k};S}$, for small q we may put

$$E_{\mathbf{k}+\mathbf{q},\mathbf{k};S}=E_{\text{pair}}, \quad (2)$$

which is independent of $\mathbf{k}$ and S.

Now let us switch on the residual interaction,

$$H'=\sum_{(ij)}V(ij), \quad (3a)$$

where

$$V(ij)=-[V_s(3+\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)})(1-\boldsymbol{\sigma}^{(i)}\cdot\boldsymbol{\sigma}^{(j)}) +V_t(1-\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)})(3+\boldsymbol{\sigma}^{(i)}\cdot\boldsymbol{\sigma}^{(j)})]\delta(\mathbf{r}_i-\mathbf{r}_j)/16. \quad (3b)$$

On the right-hand side, V_s and V_t represent the strengths of the residual interactions in the singlet and triplet spin states, respectively.

A nuclear matrix element between two $\bar{n}$-p pair states can be given by

$$F_S=\langle\mathbf{k}'+\mathbf{q},\mathbf{k}';S(S_z)|H'|\mathbf{k}+\mathbf{q},\mathbf{k};S(S_z)\rangle, =\langle\mathbf{k}'+\mathbf{q},\mathbf{k}|\delta(\mathbf{r}_1-\mathbf{r}_2)|\mathbf{k}',\mathbf{k}+\mathbf{q}\rangle \times\begin{cases}(3V_t-V_s)/2 & (S=0),\\ (V_t+V_s)/2 & (S=1).\end{cases} \quad (4)$$

In the case of a Rosenfeld mixture,

$$F_S=1.2V_t/\Omega \quad (S=0) \\ =0.8V_t/\Omega \quad (S=1), \quad (5)$$

where Ω stands for the nuclear volume $(4\pi/3)r_0{}^3A$. Since V_s and V_t are positive quantities from the definition Eq. (3b), Eq. (4) shows that the residual interactions between ($\bar{n}$-p) pairs are effectively repulsive.

When the residual interactions Eq. (4) are switched on, the most symmetric states are pushed up[11] and become collective states,

$$|\text{coll}\{\mathbf{q};S(S_z)\}\rangle=(\sqrt{2}/(N-Z)^{1/2})\sum_{\mathbf{k}}\sum_{\mu_p,\mu_n}(-)^{\frac{1}{2}-\mu_n} \times\langle\tfrac{1}{2}(\mu_p)\tfrac{1}{2}(-\mu_n)|S(S_z)\rangle a^\dagger_{\mathbf{k}+\mathbf{q},\mu_p}b_{\mathbf{k},\mu_n}|0\rangle, \quad (6a)$$

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and

$$E_{\mathrm{coll}(S)} = E_{\mathrm{pair}} + (N-Z)F_S/2 \quad (q \text{ small}). \qquad (6b)$$

It is interesting to note that, if the residual interactions are spin-independent $V_s = V_t$, we obtain $E_{\mathrm{coll}(S=0)} = E_{\mathrm{coll}(S=1)}$ as expected from the supermultiplet theory.[1]

Now we consider the beta-decay operators, which agree with $\int 1$ and $\int \boldsymbol{\sigma}$ as q goes to zero,

$$m^{(V)}(\mathbf{q}) = g_V \sum_{\mathbf{k},\mu_p,\mu_n} a^{\dagger}_{\mathbf{k}+\mathbf{q},\mu_p} b_{\mathbf{k},\mu_n} \delta_{\mu_p,\mu_n}, \qquad (7a)$$

and

$$m_M^{(A)}(\mathbf{q}) = g_A \sum_{\mathbf{k},\mu_p,\mu_n} (-)^{\frac{1}{2}-\mu_n} \sqrt{2} \langle \tfrac{1}{2}(\mu_p) \tfrac{1}{2}(-\mu_n) | 1(M) \rangle \times a^{\dagger}_{\mathbf{k}+\mathbf{q},\mu_p} b_{\mathbf{k},\mu_n}. \qquad (7b)$$

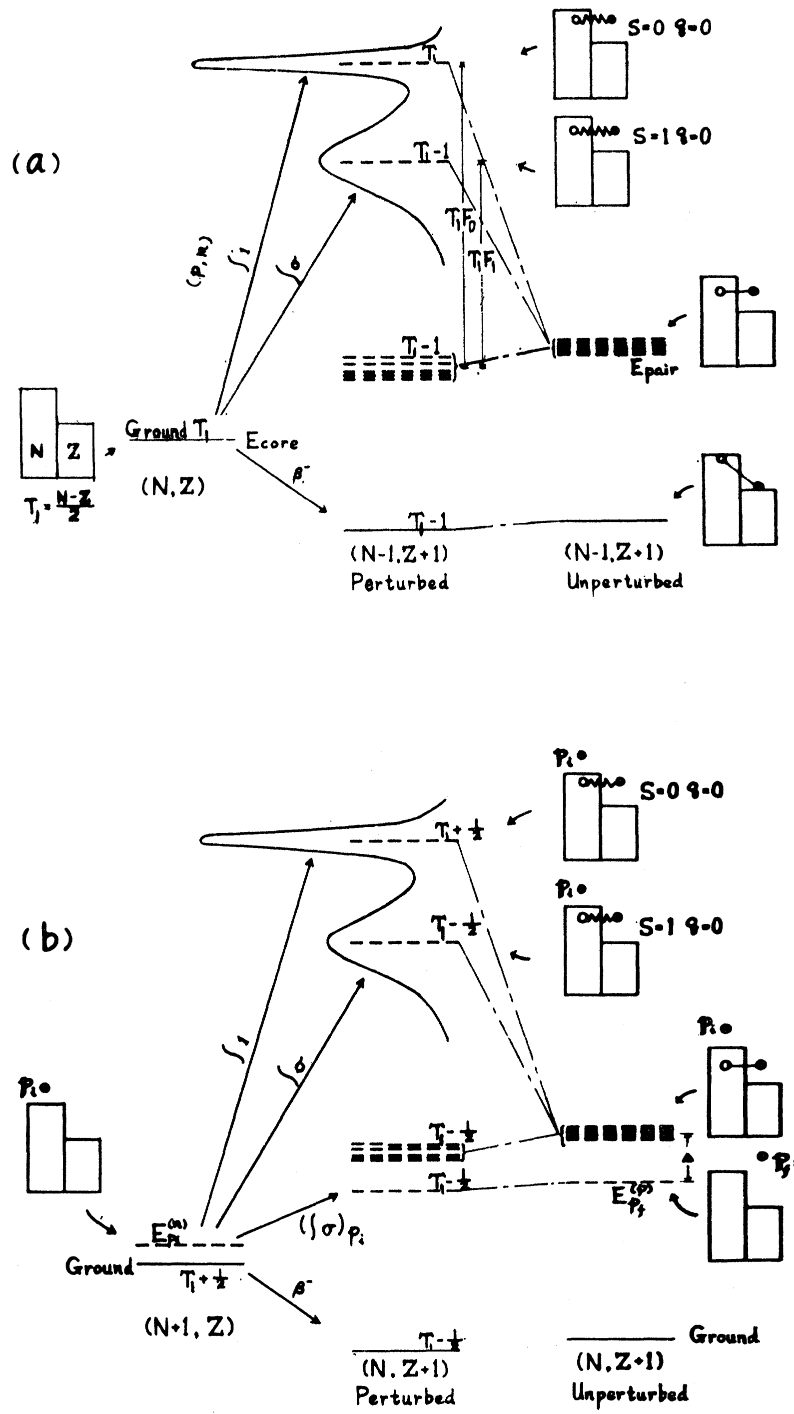

FIG. 1. Schematical picture of (p,n) reaction and beta decays. (a) and (b) correspond to the two idealized models which are discussed in Secs. 2.1 and 2.2, respectively: (a) even-even (N,Z) to odd-odd $(N+1, Z-1)$: (b) odd $(N+1, Z)$ to odd $(N, Z+1)$. T_1, $T_1+\frac{1}{2}$ etc., in the figure represent the predominant values of total isotopic spin. The effective coupling constant g_{eff} is estimated from the fictious transition amplitude $(\int\sigma)_{p_i}$ in the figure (b).

It is easily seen that the collective state with $S=0$ exhausts the beta decay sum rule for $m^{(V)}(\mathbf{q})$, while that with $S=1$ exhausts it for $m^{(A)}(\mathbf{q})$. Of course, the collective states (6a) are energetically not accessible, and can be observed only in (p-n) reactions.

It is well known that $m^{(V)}(\mathbf{q}=0)$ is only a lowering operator for the z component of total isotopic spin. Here let us briefly discuss the isospin property of the collective states. For simplicity, we will assume that neutrons and protons are confined within the same sphere. Then, the state $|0\rangle$ is an eigenstate of total isotopic spin:

$$|0\rangle=|T=T_1=(N-Z)/2,\quad T_z=T_1\rangle \tag{8a}$$

and therefore

$$\begin{aligned}|\mathrm{coll}\{\mathbf{q}=0;S=0\}\rangle&=[1/(N-Z)^{1/2}]T_-|0\rangle\\&=|T=T_1,\ T_z=T_1-1\rangle.\end{aligned} \tag{8b}$$

Because of the charge independence of nuclear forces the energy difference between $|T_1,T_1\rangle$ and $|T_1,\ T_1-1\rangle$ can be easily calculated.[9] Namely,

$$E_{\mathrm{coll}\{\mathbf{q}=0;S=0\}}=E_{\mathrm{core}}+\Delta V_c-2.5m_ec^2, \tag{9}$$

where E_{core} represents the energy of $|0\rangle$. $2.5m_ec^2$ is the neutron-proton mass difference. The single-particle Coulomb displacement ΔV_c has the magnitude of $\alpha Z/R$ approximately.

2.2. Effective Coupling Constants of Beta Decay

Now we are at the stage to discuss the beta decay of an outside nucleon. Using the notation, which was introduced in the beginning of this section, we can express the initial state as

$$|i\rangle=b^\dagger{}_{\mathbf{p}_i,\mu_{ni}}|0\rangle. \tag{10}$$

On the other hand, we consider the small admixture of the collective state of the core in the final state;

$$\begin{aligned}|f\rangle=C_fa^\dagger{}_{\mathbf{p}_i+\mathbf{q},\mu_{pf}}|0\rangle+\sum_{\mathbf{k}}C_{\mathbf{k},\mathbf{q}}{}^Sb^\dagger{}_{\mathbf{p}_i,\mu_{ni}}\\ \times|\mathbf{k}+\mathbf{q},\mathbf{k};S(\mu_{pf}-\mu_{ni})\rangle.\end{aligned} \tag{11}$$

All the other excited states, which have nothing to do with the beta decay under consideration, are omitted. This procedure is expected to be good insofar as C_f is close to 1. The unperturbed energies of $a^\dagger{}_{\mathbf{p}_f,\mu_{pf}}|0\rangle$, $b^\dagger{}_{\mathbf{p}_i,\mu_{ni}}|0\rangle$, and $b^\dagger{}_{\mathbf{p}_i,\mu_{ni}}|\mathbf{k}+\mathbf{q},\mathbf{k};S(\mu_{pf}-\mu_{ni})\rangle$ are written as

$$E_{\mathbf{p}_f}{}^{(p)},\quad E_{\mathbf{p}_i}{}^{(n)}\quad\text{and}\quad E_{\mathbf{p}_i}{}^{(n)}+E_{\mathrm{pair}}-E_{\mathrm{core}}.$$

If we put

$$\Delta=E_{\mathbf{p}_i}{}^{(n)}+(E_{\mathrm{pair}}-E_{\mathrm{core}})-E_{\mathbf{p}_i+\mathbf{q}}{}^{(p)},$$

the Δ can be understood as the pairing energy of the core neutrons when q is small.

Now, writing $b^\dagger{}_{\mathbf{p}_i,\mu_{ni}}|0\rangle$ as $|0\rangle'$ and adding a prime to every relevant state, we can rewrite Eq. (11) as

$$\begin{aligned}|f\rangle=C_f\sum_S(-)^{\frac12-\mu_{ni}}\langle\tfrac12(\mu_{pf})\tfrac12(-\mu_{ni})|S(\mu_{pf}-\mu_{ni})\rangle\\ \times|\mathbf{p}_i+\mathbf{q},\mathbf{p}_i;S(\mu_{pf}-\mu_{ni})\rangle'\\ +\sum_{\mathbf{k},S}C_{\mathbf{k},\mathbf{q}}{}^S|\mathbf{k}+\mathbf{q},\mathbf{k};S(\mu_{pf}-\mu_{ni})\rangle'.\end{aligned} \tag{12}$$

It is clear that we may treat $S=0$ and 1 separately because our residual interactions are given by Eq. (3). Therefore our model includes one isolated level and T_1 degenerate ones for each S in the unperturbed state.

Now, let us restrict ourselves to the case of $q=0$, corresponding to the allowed transitions. It is clear that

$$b^\dagger{}_{\mathbf{p}_i,\mu}|0\rangle=|T_1+\tfrac12,T_1+\tfrac12\rangle, \tag{13a}$$

$$b^\dagger{}_{\mathbf{p}_i,\mu}|\mathbf{k},\mathbf{k};1(S_z)\rangle=|T_1-\tfrac12,T_1-\tfrac12\rangle, \tag{13b}$$

$$\begin{aligned}(1/(2T_1+1)^{1/2})a^\dagger{}_{\mathbf{p}_i,\mu}|0\rangle+((2T_1)^{1/2}/(2T_1+1)^{1/2})b^\dagger{}_{\mathbf{p}_i,\mu}\\ \times|\mathrm{coll}\{q=0,S=0\}\rangle=|T_1+\tfrac12,T_1-\tfrac12\rangle,\end{aligned} \tag{13c}$$

and

$$\begin{aligned}((2T_1)^{1/2}/(2T_1+1)^{1/2})a^\dagger{}_{\mathbf{p}_i,\mu}|0\rangle-(1/(2T_1+1)^{1/2})b^\dagger{}_{\mathbf{p}_i,\mu}\\ \times|\mathrm{coll}\{q=0,S=0\}\rangle=|T_1-\tfrac12,T_1-\tfrac12\rangle.\end{aligned} \tag{13d}$$

Since we are interested only in the lowest perturbed level, Eq. (12) can be written as

$$\begin{aligned}|f\rangle=C_f\sum_S(-)^{\frac12-\mu_{ni}}\langle\tfrac12(\mu_{pf})\tfrac12(-\mu_{ni})|S(\mu_{pf}-\mu_{ni})\rangle\\ \times|\mathbf{p}_i+\mathbf{q},\mathbf{p}_i;S(\mu_{pf}-\mu_{ni})\rangle'+(T_1)^{1/2}\sum_S C^S\\ \times|\mathrm{coll}\{q=0;S(\mu_{pf}-\mu_{ni})\}\rangle'.\end{aligned} \tag{14}$$

When each unperturbed state is required to have a definite total isospin, the most important consequence is that $C^{S=0}=0$. Small corrections[10] might arise from the admixture due to Coulomb forces, which we neglect in this paper. Equation (13d) tells us that the use of (14) as an approximate eigenstate of total isospin is justified for the part of $S=1$ when $(2T_1)^{1/2}$ is large in comparison with 1.

By solving the 2×2 secular equation, we obtain

$$\begin{aligned}(T_1)^{1/2}C^{S=1}/\{C_f(-)^{\frac12-\mu_{ni}}\langle\tfrac12(\mu_{pf})\tfrac12(-\mu_{ni})|1(\mu_{pf}-\mu_{ni})\rangle\}\\ =-(T_1)^{1/2}F_1/(T_1F_1+\Delta+\epsilon),\end{aligned} \tag{15a}$$

where

$$\begin{aligned}2\epsilon=[(T_1+1)F_1+\Delta]\\ -[\{(T_1-1)F_1+\Delta\}^2+4T_1F_1{}^2]^{1/2}.\end{aligned} \tag{15b}$$

The right-hand side of Eq. (15a) agrees with the usual first-order perturbation results $(T_1)^{1/2}F_1/\Delta$, when $T_1F_1\ll\Delta$. On the other hand, as T_1F_1 becomes larger than Δ, the factor is changed into $(T_1)^{1/2}F_1/(T_1F_1+\Delta)$.

From Eqs. (7b), (10), (14), and (15) we obtain

$$\begin{aligned}\langle f|m_M{}^{(A)}(q=0)|i\rangle=\sqrt{2}g_A(-)^{\frac12-\mu_{ni}}\\ \times\langle\tfrac12(\mu_{pf})\tfrac12(-\mu_{ni})|1(M)\rangle f,\end{aligned} \tag{16a}$$

where the hindrance factor f is given by

$$f=1-T_1F_1/(T_1F_1+\Delta+\epsilon)$$
$$=(\Delta+\epsilon)/(T_1F_1+\Delta+\epsilon). \quad (16b)$$

When F_1 is zero, apparently no hindrance occurs, $f=1$. On the other hand, f goes to zero in the limit of large F_1T_1/Δ. The latter result is quite reasonable, because in this limit the discrimination between the outside and the core nucleons disappear and the beta-decay strength is exclusively concentrated at the uppermost collective states.

The numerical values of f are shown in Fig. 2 as a function of $T_1=(N-Z)/2$ and F_1/Δ. If we put $T_1F_1=\Delta$, $f=\frac{1}{2}$ is obtained within the error of order $1/T_1$. Since $T_1F_1>\Delta$ in the actual case, the first-order perturbation gives misleading results.

3. DISCUSSIONS AND COMPARISON WITH EXPERIMENTAL DATA

Let us consider what kind of modifications will be necessary if we adopt more realistic models.

The next simplest model is to assume that the nuclear core is made of $L-S$ doubly closed shells. Then an unperturbed $\bar{n}$-p pair state is written as

$$|L,S,J,J_z=0\rangle$$
$$=\sum_{m_1,m_1',\mu_1,\mu_1',M}(-)^{l_1'-m_1'+\frac{1}{2}-\mu_1'}\langle l_1(m_1)l_1'(-m_1')|L(M)\rangle$$
$$\times\langle\tfrac{1}{2}(\mu_1)\tfrac{1}{2}(-\mu_1')|S(-M)\rangle\langle L(M)S(-M)|J(0)\rangle$$
$$\times a^\dagger_{l_1m_1\mu_1}b_{l_1'm_1'\mu_1'}|0\rangle, \quad (17)$$

which has the orbital, spin, and total angular momenta, L, S, and J, respectively. The nuclear matrix element of residual interaction Eq. (3) can then be expressed as follows after several standard manipulations;

$$F(LSJ)$$
$$=\langle LSJJ_z|H'|LSJJ_z\rangle$$
$$=F^0(l_1l_2l_1'l_2')\{(2l_1+1)(2l_2+1)(2l_1'+1)(2l_2'+1)\}^{1/2}$$
$$\times(2L+1)^{-1}(l_10l_1'0|L0)(l_20l_2'0|L0)$$
$$\times\begin{cases}(3V_t-V_s)/2 & (S=0)\\ (V_t+V_s)/2 & (S=1),\end{cases} \quad (18a)$$

where

$$F^0(l_1l_2l_1'l_2')=\frac{1}{2}\int R^*_{n_1l_1}(r)R^*_{n_2l_2}(r)R_{n_1'l_1'}(r)$$
$$\times R_{n_2'l_2'}(r)r^2dr. \quad (18b)$$

In this model, the nuclear matrix elements are not the same simple constant any more, and they depend on the relevant states. After averaging[11] over states, F^0 can be regarded as a constant depending only on L, S, and J.

Another important assumption was that all the unperturbed state of core $\bar{n}$-p pairs are degenerate. It is easily seen from the $j-j$ coupling level scheme that the assumption of degeneracy is good[5] only in the case of an isobaric state ($L=S=0$). The $\bar{n}$-p state with $L=0$ and $S=1$ are distributed over the region of several MeV because of the spin-orbit splitting. Furthermore, only the isobaric state[9] is known to be a well-defined eigenstate with width of order 100 keV.[8] The other possible collective states ($S\neq0$ or $L\neq0$) must be understood as broad resonance states similar to the giant peak in nuclear photoeffect.

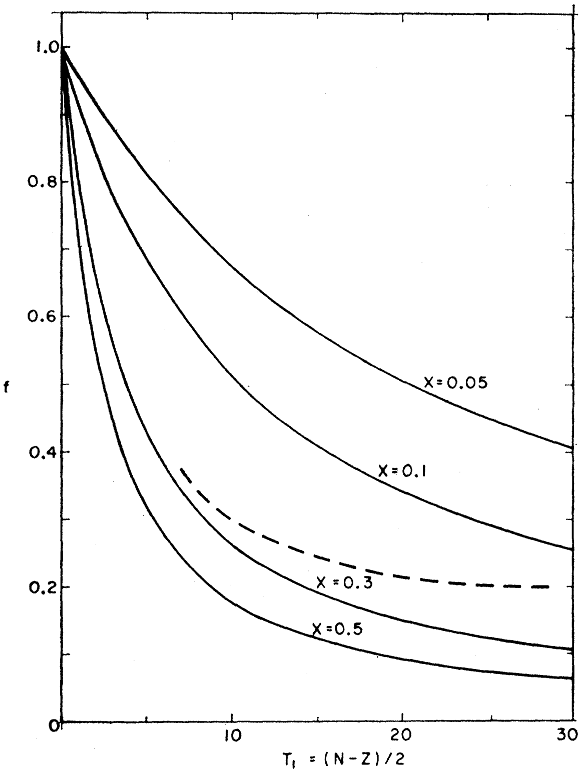

FIG. 2. The hindrance factor f of Eq. (16b) in the Fermi gas model is shown as a function of $T_1=(N-Z)/2$ and $X=F_1/\Delta$, where F_1 is given by Eq. (4) and Δ stands for the average pairing energy of relevant neutrons. The broken line in this figure corresponds to the "theoretical curve" in Fig. 3, which was obtained by the procedure described in Sec. 3.

Now, it is apparent that, even if we start from more realistic models, we should get an expression for the effective coupling constant, similar to (16b), assuming we may regard the relevant $\bar{n}$-p level as approximately degenerate and the relevant nuclear matrix elements of residual interactions as some constant.

In this semiphenomenological spirit, we would like to look at the experimental $\log f_0t$ values. First it is interesting to compare the weighted average of F^0 in shell model with the corresponding quantity $2\pi/\Omega=(0.87/A)10^{39}$ cm^{-3}. Using the wave functions of harmonic oscillator potential, we have

$$\langle F^0(l_1l_2l_1l_2)\rangle_{\rm av}=(0.494/A)10^{39}\ {\rm cm}^{-3}, \quad (19)$$

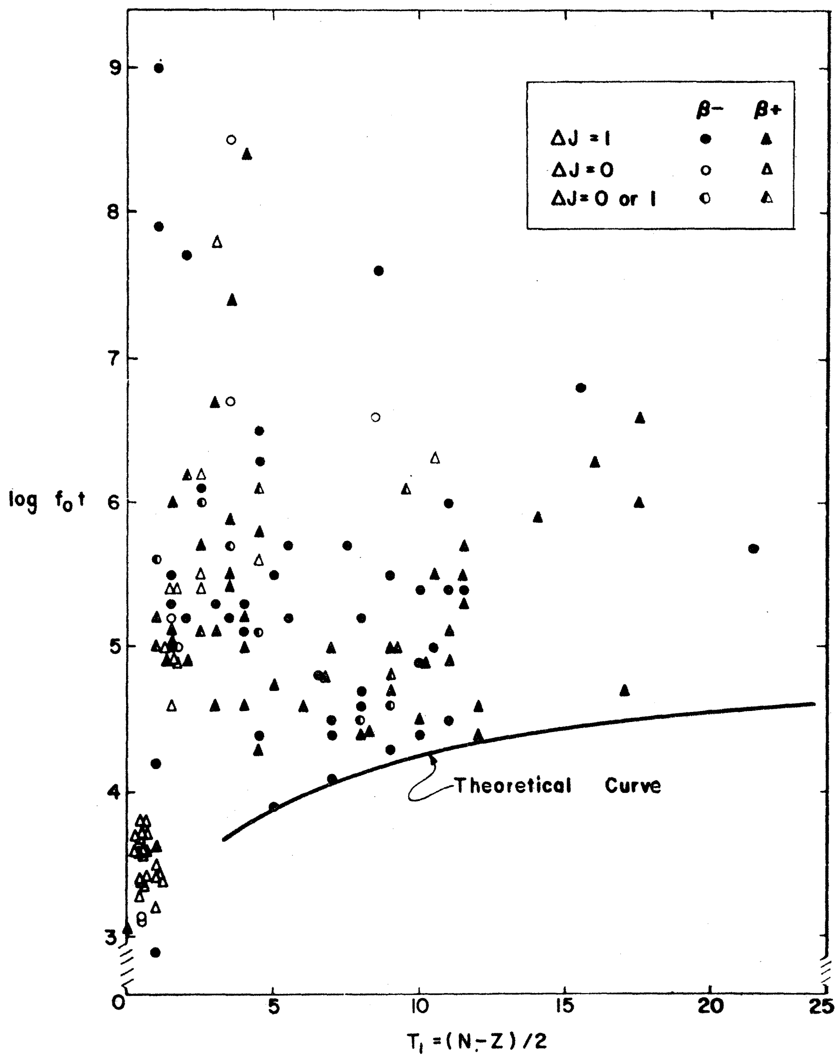

FIG. 3. Empirical $\log f_0 t$ values of the ground-ground allowed beta transitions, which were brought together from Nuclear Data Sheets. (Ref. 6) $T_1=(N-Z)/2$ corresponds to the parent (daughter) nucleus for β^- (β^+) decay in accordance with our model. The theoretical curve in the figure was obtained by the following assumptions; $\log f_0 t = 3.72 - \log 3 f^2$, where f is the calculated hindrance factor. In Sec. 3 the reason is discussed why our theoretical curve seems to give the lower limit of empirical $\log f_0 t$ values in the figure.

which gives a reasonable value[12] for Pb. Therefore, the difference between the Fermi gas and the usual shell model is less than factor 2 for F^0.

Another restriction on F^0 is obtained from Eqs. (6b) and (9);

$$(N-Z)F_0/2 = E_{p_i}{}^{(n)} - (E_{p_i}{}^{(p)} + 2.5 m_e c^2) + (\Delta V_c - \Delta). \quad (20)$$

Using the relation

$$E_{p_i}{}^{(n)} - (E_{p_i}{}^{(p)} + 2.5 m_e c^2) > 0,$$

we obtain

$$(N-Z)F_0/2 < \Delta V_c - \Delta.$$

In Fig. 3 the empirical $\log f_0 t$ values of the ground-ground allowed beta transitions are shown as a function of $T_1=(N-Z)/2$. In the figure a theoretical curve is drawn. The curve was obtained by the following procedures; assuming that the residual interaction is a Rosenfeld mixture, we get from Eqs. (5) and (19).

$$F_1 = (2/3)F_0 = (2/3)(V_t/4\pi)2\langle F^0\rangle_{\rm av} = 61.4/A \text{ MeV},$$

where

$$V_t = 77.7\times10^{-39} \text{ MeV cm}^3$$

is assumed.[12] This value of V_t is known from our previous analysis[5] to satisfy the above restriction, $(N-Z)F_0/2 < \Delta V_c - \Delta$. For the numerical value of Δ, the empirical values of pairing energies for neutrons[13] were taken,

$$\Delta = 2\times 11.56/N^{0.552} \text{ MeV}.$$

The choice of Δ is a little ambiguous, because in our model Δ should correspond to the averaged pairing

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energies of relevant neutrons. Also, the approximate relation $N=1.71Z-14.2$ was used.[5] Now by using Eq. (16b) we can calculate the hindrance factor f and the estimates of $\log f_0t$ values are obtained[14]; $D=f_0t \times|\int\boldsymbol{\sigma}|^2$, where $\log D=3.72$ and $|\int\boldsymbol{\sigma}|^2=3f^2$. Our theoretical curve is a sort of Weisskopf estimate, which neglects all the individuality of the nucleus but takes into account only the nuclear core polarization.

It is interesting to note that our theoretical curve seems to give the lower limit of empirical $\log f_0t$ values in Fig. 3. This is consistent with the idea that there exists the hindrance effect due to the nuclear core polarization in heavy nuclei, since the other effects are believed to have tendency of increasing f_0t values but smaller magnitude than required.

It must be remembered that in our Fermi gas model the final state with momentum $p_f=p_i$ is not the ground state of our model. Namely, it becomes an important problem to what extent this higher momentum state is contained in the actual final-state wave function. Originally, the beta-decay operators for allowed transitions are those which select components with the same momenta in initial and final states. In order to discuss this overlapping effect, the individual transition must be studied in detail. Already the overlapping effects have been studied; (a) l-forbidden transitions, (b) overlapping[2] of the nuclear core in the initial and final states, (c) the particle-phonon coupling, (d) the blocking effect in the pairing model,[3,4] and so on.

Unfortunately it seems difficult to study only the effect due to the nuclear core polarization. However, as was shown in this paper, the hindrance factor f would be a smooth function of $(N-Z)$ and A. In order to determine the magnitude of the hindrance factor f experimentally, we should statistically investigate the $\log f_0t$ values of heavy nuclei by introducing the phenomenological coupling constant $g_{\text{eff}}=fg_A<g_A$ instead of g_A. When the relative f_0t values will be thoroughly understood, the magnitude of the core polarization effects will be exactly known as well.

This can be easily extended to the case of forbidden transitions,[15] especially straightforwardly to the second-forbidden transitions. In the case of first- or third-forbidden ones, situation is more complicated because $\bar{n}$-p pairs must have odd parity. The importance of $\bar{n}$-p correlations has previously been suggested[7] in the case of RaE.

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